Influence of triaxiality (damage in polymers)

Transition from plane stress PS (thin specimen or close to the surface) to plane strain PE (thick specimen or on symmetry plane).
The smaller plastic zone size in plane strain results from the high triaxiality of the stress state, which restricts plastic yielding (Tr ε^{pl} =0).

$$\eta = \frac{Tr\sigma}{3\sigma_{VM}} \quad \text{assuming} \;\; \nu = 1/3:$$

$$= 8/3 \;\; \text{in PE}$$

$$= 2/3 \;\; \text{in PS}$$

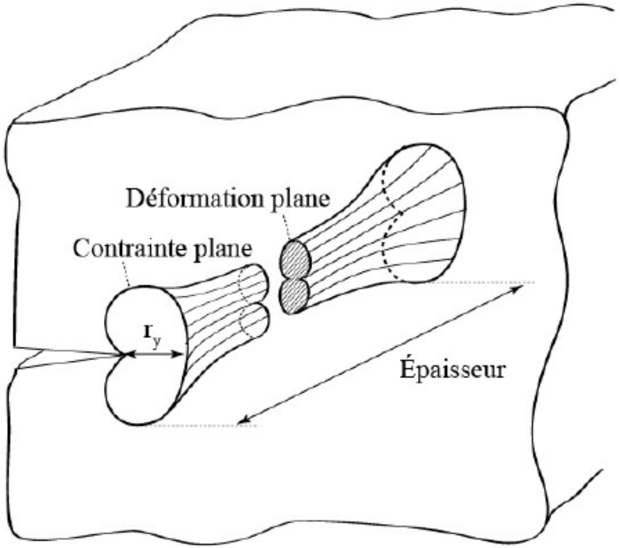
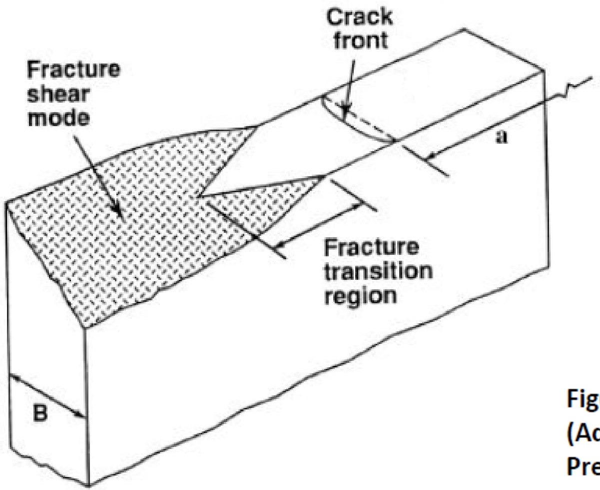


Figure 4-3. Sketch of the general shape of a Mode I crack tip plastic zone across a thick plate (from Janssen, Zuidema, Wanhill, Fracture Mechanics, 2nd Edition, CRC Press, London, 2014).

Influence of plastic zone size (metals)

Mind the unit of KI !
Ksi-inch$^{1/2}$



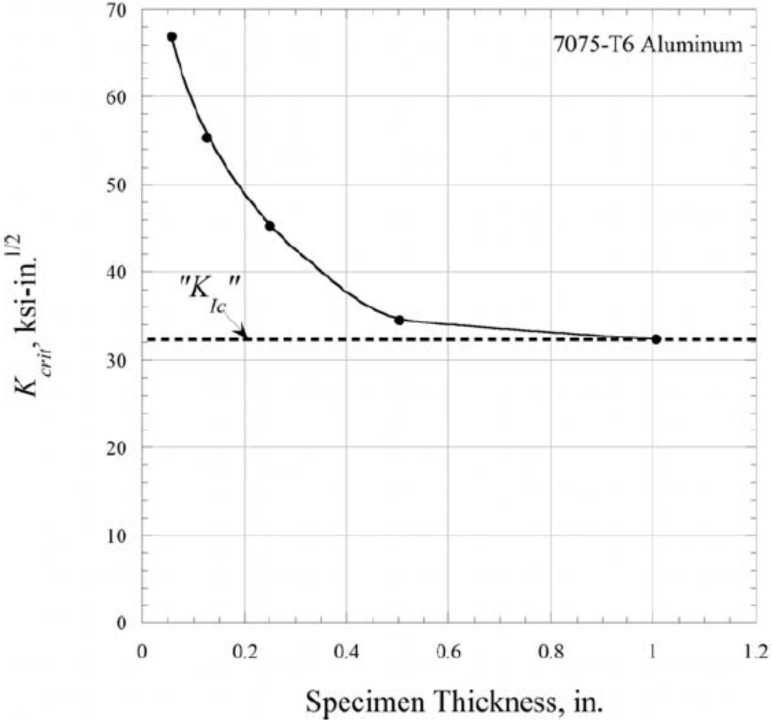


Figure 4-8. Variation of measured fracture toughness with specimen thickness for 7075-T6 Aluminium. (Adapted by Anderson from Barsom and Rolfe, Fracture and Fatigue Control in Structures. 2nd Ed., Prentice-Hall, Englewood Cliffs, NJ, 1987).

Figure 4-7. Typical appearance of a fracture surface where initial crack extension under plane strain conditions is superseded by fracture under plane stress.

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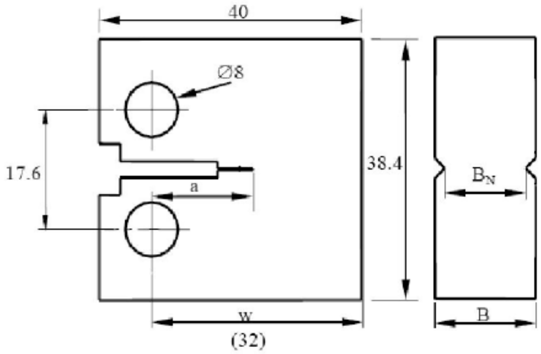
Conditions for valid K_{Ic} testing

When performing laboratory plane strain mode I K_{Ic} tests on standard specimens, the following empirical size requirements have been adopted to ensure reproducible results for different elastic-plastic materials:



$$a, B, (W-a) \geq 2.5\left(\frac{K_{Ic}}{\sigma_y}\right)^2$$

The minimum requirements for *a* and *W* – *a* ensure that the plastic zone is sufficiently small for fracture to be *K*-controlled (20 to 50 times the plastic zone size). The requirement for *B* is intended to ensure plane strain conditions along the crack front, although it is often far more stringent than necessary.

Linear superposition principle

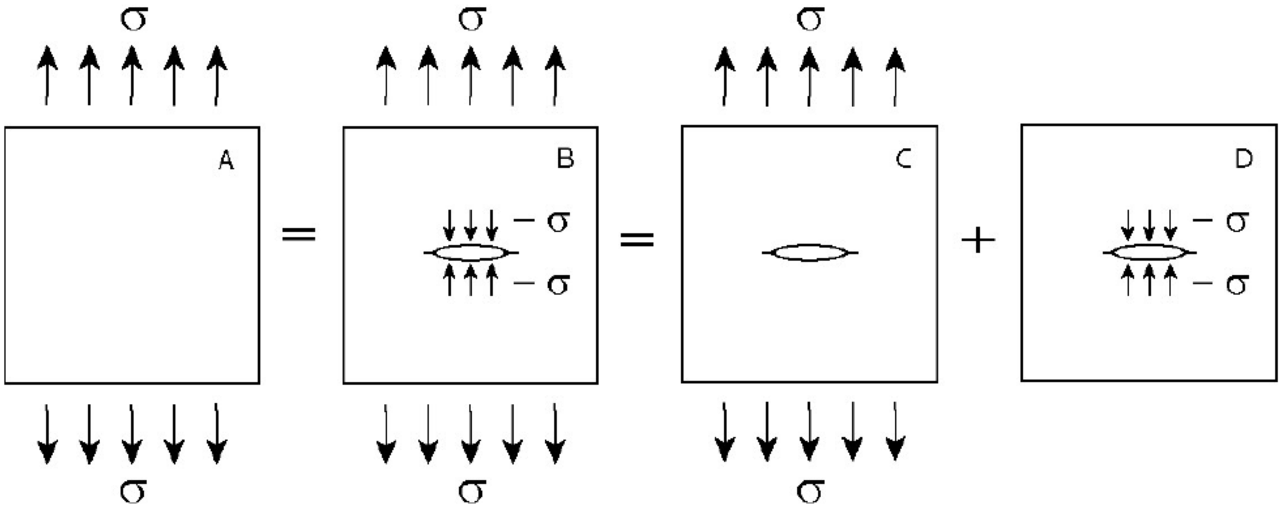


Figure 2-6. Using the linear superposition principle to deduce that the stress intensity factor corresponding to a crack loaded by a uniform stress σ applied to the crack planes in an otherwise stress-free solid, which we shall call K_x, is the same as the stress intensity factor given by the same stress applied remotely to the whole of the specimen (case C above). In A, the stress is uniform everywhere. To obtain this situation when a remote stress σ is applied in the presence of a crack, we need to apply an additional negative stress σ to the crack faces (case B). In case B, then, $K_B = 0$. If we do not apply this negative stress to the crack faces (case C), then the stress intensity factor will take some finite value K_c corresponding to the crack length. By superposition, case B is equivalent to case C + case D, and hence $K_B = 0 = K_c + K_D$. By inverting the direction of the stress in case D we see that $K_x = -K_D$. Therefore $K_c - K_x = 0$ and $K_c = K_x$, QED.

Linear superposition principle

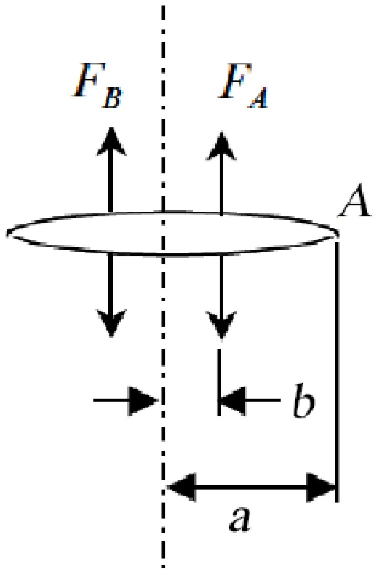


Figure 2-7. Crack with forces F_A and F_B applied at distances *b* and *a* + *b* from the tip A of a through-thickness crack of length 2*a* in an infinite plate of thickness *B*.

Now let's look at another example, starting from Figure 2-7. For this situation, it is possible to show (you can look up how in A.A. Wells, Br. Weld. J. 12, 1965, 2, If you're interested) that the stress intensity at crack tip A due to (linear) force F_A acting at a distance $x = b$ from the crack tip is

$$K_{IA} = \frac{F_A}{B(\pi a)^{1/2}} \sqrt{\frac{a+b}{a-b}} \qquad \text{(2-31)}^1$$

and that due to force F_B, which acts at a distance $x = a + b$ from the crack tip A, is

$$K_{IB} = \frac{F_B}{B(\pi a)^{1/2}} \sqrt{\frac{a-b}{a+b}} \qquad \text{(2-32).}$$

By superposition, then, if the two forces are of equal magnitude, *F*

$$K_I = K_{IA} + K_{IB} = \frac{2F}{B\pi^{1/2}} \sqrt{\frac{a}{a^2-b^2}} \qquad \text{(2-33.}$$

Hence, if we consider a continuous symmetrical distribution of forces along the crack such the *F* at any position along the crack is given by $B\sigma(x)dx$, we obtain

$$K = 2\sqrt{\frac{a}{\pi}}\int_0^a \frac{\sigma(x)}{\sqrt{a^2-x^2}}dx \qquad \text{(2-34)}$$

The Dugdale strip yield model

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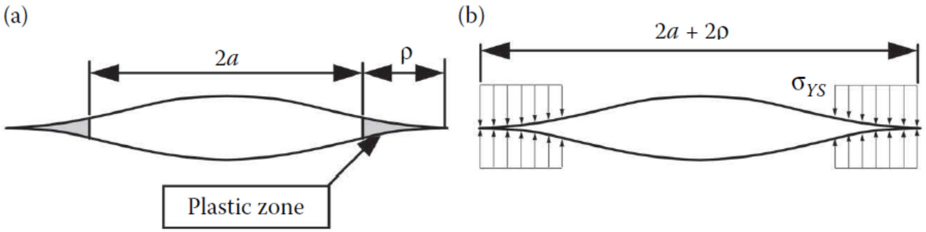

FIGURE 2.31
The strip yield model. The plastic zone (a) is modeled by yield magnitude compressive stresses at each crack tip (b).

$$K_{closure} = -2\sigma_{YS}\sqrt{\frac{a+\rho}{\pi}}\int_a^{a+\rho}\frac{dx}{\sqrt{(a+\rho)^2 - x^2}}$$

Solving this integral gives

$$K_{closure} = -2\sigma_{YS}\sqrt{\frac{a+\rho}{\pi}}\cos^{-1}\left(\frac{a}{a+\rho}\right) \qquad (2.86)$$

The stress intensity from the remote tensile stress, $K_\sigma = \sigma\sqrt{\pi(a+\rho)}$, must balance with $K_{closure}$. Therefore,

$$\frac{a}{a+\rho} = \cos\left(\frac{\pi\sigma}{2\sigma_{YS}}\right) \qquad (2.87)$$

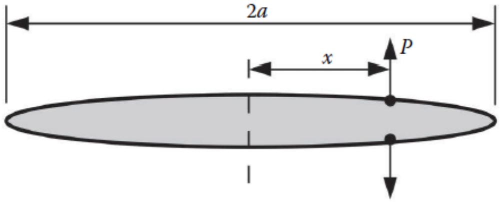

FIGURE 2.32
Crack opening force applied at a distance x from the centerline.

$$\rho = \frac{\pi^2\sigma^2 a}{8\sigma_{YS}^2} = \frac{\pi}{8}\left(\frac{K_I}{\sigma_{YS}}\right)^2 \qquad r_p = \frac{1}{\pi}\left(\frac{K_I}{\sigma_{YS}}\right)^2$$

Fatigue fracture

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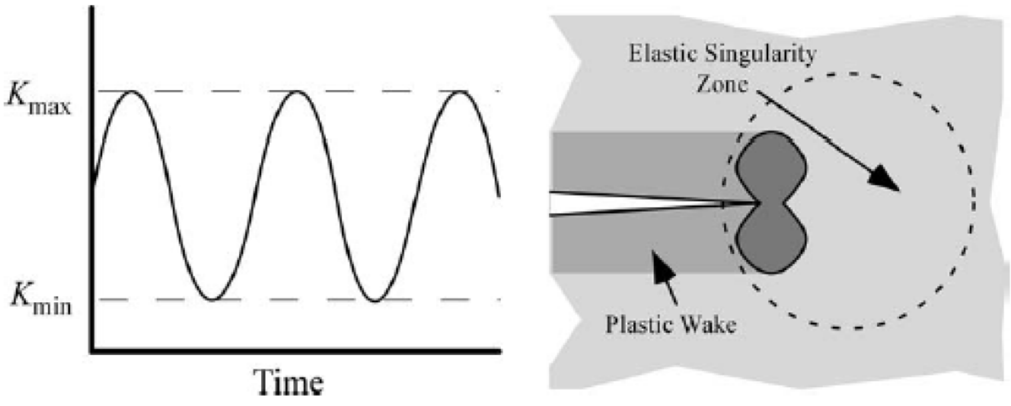

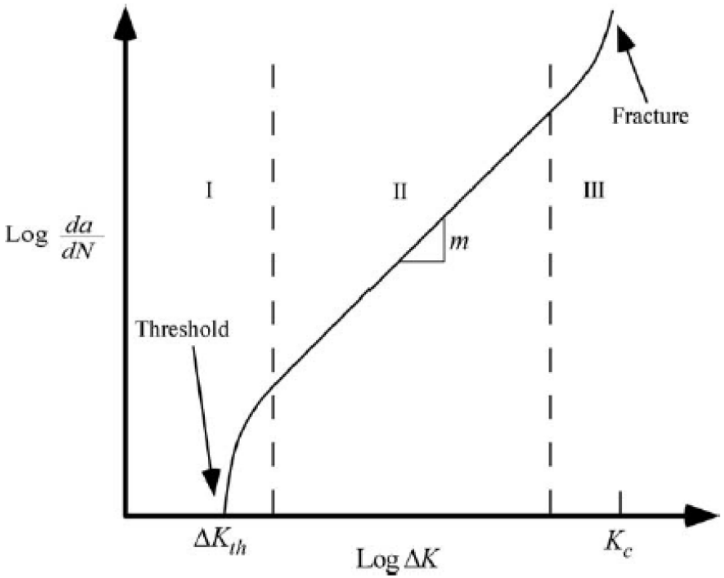

Small scale yielding condition and formation of a plastic wake

Typical crack growth in metals

Paris law : $\frac{da}{dN} = C\Delta K^{m}$ with m between 2 and 4.

Fracture mechanics
Last class on May 26th:
- CTOD: Crack tip opening displacement
- Recap and Q&As

- Fracture mechanics, T.L. Anderson
- Available at EPFL library and online
  - Be sure to be connected to VPN for accessing online version
  - https://www.epfl.ch/campus/library/